Analysis of unidirectional coupling in topological valley photonic crystal waveguides

Wen-Sheng Ruan, Xin-Tao He, Fu-Li Zhao, and Jian-Wen Dong

Abstract—Valley photonic crystal is a typical strategy of topological photonics with promising applications for novel integrated waveguides. The valley pseudospin provides an intrinsic paradigm to implement unidirectional coupling with robustly optical transport. But the key issue on the analysis of unidirectional coupling based on topological modes is lack of deep understanding on valley photonic crystals. In this work, we systematically study the mechanism and performance of unidirectional coupling between valley photonic crystal waveguides and chiral dipole emitters. Forming by two topologically-distinct valley photonic crystals, there are two types of valley-interface waveguides, i.e. bearded stack with glide-plane symmetry and zigzag stack with inversion symmetry. Analytical derivation shows that the Stokes parameters of topological eigenmodes can predict the directionality in the above two types of valley-interface waveguides, which is also confirmed by full-wave simulations. The strategy of bearded valley-interface waveguide can be easier to achieve high efficiency of unidirectional coupling than the zigzag case, and can relax the bias error of frequency and source location. It is also proved that the topological-protected unidirectional coupling supports robust transport against sharp-bending interface. Finally, we give a brief summary and perspective in the development of topological integrated photonics. This work gives a deterministic guidance on chiral light-matter interaction based on topological photonics, as well as shows the outlook of topological integrated photonics.

Index Terms—Optical waveguides, photonic crystals, topological photonics, unidirectional coupling, valley degree of freedom.

I. INTRODUCTION

Optical coupling between dipole emitters and nanophotonic structures is one of essential mechanisms to engineer light-matter interaction at nanoscale. With local polarization (‘spin’) and propagation direction (‘orbit’) locking, unidirectional coupling (UDC) by chiral dipole emitters provides a new approach to control intriguing light manipulation and complex information processing. Recently, exploration of UDC has attracted extensive attentions in various optical waveguides, leading the emerging fields of chiral nanophotonics and chiral quantum optics [1]–[3]. By means of evanescent tails of waveguide modes, such as in rectangular/circular-cross-section nanowires and plasmonic surfaces, one can apply the intrinsic transverse-spin-direction locking property to implement chiral UDC [1], [2], [4]–[12]. Moreover, line-defect photonic crystal waveguides (PCWs) provide an extrinsic way for high-efficiency chiral UDC [13]–[15]. Through flexible control of the line defects, it is accessible to localize mode energy around the circular-polarized points (C-points), which is the crucial role to improve UDC efficiency in PCW.

Recent advances of topological photonic crystals [16]–[20] render a new degree of freedom (DoF), i.e. pseudospin, to manipulate the optical coupling in nanophotonic structures. The phenomena of UDC between chiral sources and topologically optical modes have been demonstrated in integrated-compatible PCWs, analogous to quantum spin- and valley-Hall topological phases [21]–[28]. Since such pseudospin DoF is highly relevant to chiral dipole moment, the implement of chiral UDC is essentially derived from pseudospin-chirality locking. Through the control of chirality of dipole moment, the pseudospin-dependent forward edge states can be selectively excited when the pseudospin-flip backward modes are naturally suppressed [29]–[31]. In addition to in-plane backward coupling, another key factor to improve UDC efficiency is the suppression of non-guided modes [32], particularly for the out-of-plane radiation. Benefitting from the operation of below-light-cone band structures, some very recent experiments on valley photonic crystals (VPCs) show outstanding ability to explore high-efficiency UDC in topological integrated photonics [28] and quantum optics [33], [34]. It is in need of systematical analysis on unidirectional coupling mechanism, which is still lack of deep understanding on valley photonic crystals.

In this work, we systematically study the UDC mechanism between chiral dipole emitters and VPC waveguides, in terms of eigenmode analysis and full-wave simulations. Two topologically-distinct valley photonic crystals with opposite sign of valley Chern indices have been designed. Forming by such two domain walls, there are two typical types of valley-interface waveguides, i.e. bearded-stack waveguide with glide-plane symmetry and zigzag-stack one with inversion symmetry. For comparison between these two types of waveguides, the
UDC performance is quantified by using both directionality $D$ and unidirectional coupling efficiency $\beta$. Based on analytical derivation, the directionality of light propagation along both waveguides are consistent with the Stokes parameter of eigenmodes. This is also confirmed by numerical calculations that near-unity $D$ can be realized by placing the chiral dipole emitter at the C-points of eigenmodes. In addition, to obtain high coupling efficiency, one should promote the match of C-point and the field energy of eigenmode, highly related to the symmetry of waveguides. Simulation results show that bearded-stack waveguide is accessible for higher coupling efficiency than that of zigzag-stack waveguide, although the directionality for both of them is near unity. Finally, we discuss the robustness of light propagation along bearded-stack VPC waveguide and line-defect PCW. The former one can maintain high directionality over broadband even along 120°-bending interface due to valley-Hall topological phase, while the directionality of latter one decreases dramatically.

II. GENERAL ANALYSIS METHOD FOR UNIDIRECTIONAL COUPLING

The interaction between dipole emitters and optical structures could be enhanced by local state of density (LDOS) by Purcell effect [35], [36]. The output optical power at the position of dipole is proportional to the LDOS of coupling, i.e. 

$$P(r', \omega) \propto \text{Im} \left\{ \mathbf{d} \cdot \mathbf{G}(r, r'; \omega) \cdot \mathbf{d} \right\}, r = r'$$ \hspace{1cm} (1)

where $\mathbf{d}$ is the complex dipole moment of the point emitter. $\mathbf{G}(r, r')$ is the Green's function related to the electric field response at position $r$ excited by a dipole emitter at $r'$ [36].

Consider a quasi-1D waveguide that the guided modes propagate along $x$ direction, Green's function can be decomposed into two components, the major part of guided modes $G^{GD}$ and the minor part of other modes $G^{O}$ which is dominant for the continuous radiation modes $G^{GD}$ in general. In the bandgap of PCW, the slab modes are suppressed and almost power of the dipole emitter couples to the guided modes and continuous radiation modes. For simplification below, we assume that the in-plane propagating waves are contributed by the coupling of guided modes and the optical power scattering into other directions is contributed by the coupling of out-of-plane radiation. In PCWs, the Green's function of guided mode $G^{GD}$ can be written as the following form [15], [37], [38],

$$G^{GD}(r, r'; \omega) = i \frac{\alpha \omega}{2 v_g} \left\{ \Theta(x-x') f_{k_x}(r) f_{k_x}^*(r') \exp \left[ ik_x(x-x') \right] \right. \right.

+ \left. \Theta(x'-x) f_{k_x}(r) f_{k_x}^*(r') \exp \left[ -ik_x(x-x') \right] \right\},$$ \hspace{1cm} (2)

where $\alpha$ and $v_g$ are the lattice constant and group velocity of PCW, respectively. $\Theta$ is the Heaviside step function. $f_{k_x}(r)$ is the electric fields for Bloch mode of PCW with wavevector $k_x$. Here we define $+k_x$ (-$k_x$) is the rightward (leftward) direction. For $r = r'$, substituting (2) into (1), we can get

$$P^{GD}(r', \omega) \propto \frac{\alpha \omega}{4 v_g} \left\{ |d^* \cdot f_{k_x}(r')|^2 + |d^* \cdot f_{k_x}^*(r')|^2 \right\},$$ \hspace{1cm} (3)

From (3), the optical power coupling to guided modes can be divided into rightward part $P^R$ and leftward part $P^L$. For time-reversal symmetry (TRS), we have

$$P^R(r', \omega) \propto \frac{\alpha \omega}{4 v_g} |d^* \cdot f_{k_x}(r')|^2,$$ \hspace{1cm} (4)

$$P^L(r', \omega) \propto \frac{\alpha \omega}{4 v_g} |d^* \cdot f_{k_x}^*(r')|^2,$$

For unidirectional coupling, the dipole emitter should individually couple to the rightward or leftward guided modes and entirely suppress the coupling along the other direction, e.g. rightward UDC that $P^L \neq 0$ and $P^R = 0$.

To characterize the UDC performance, a general and primary quantity is the directionality $D$ defined as

$$D = \frac{P^R - P^L}{P^R + P^L},$$ \hspace{1cm} (5)

$D$ is the contrast ratio of optical power coupling to rightward and leftward modes. The extreme cases of $D = \pm 1$ represent ideal rightward or leftward UDC. To do this, one should generally consider a point emitter with chiral dipole moment $d = (x \pm iy)/\sqrt{2}$. Only consider the TE-like modes at $z = 0$ plane of PCW, the electric fields $f_{k_x}$ could be simplified as in-plane components, i.e. $f_{k_x}(r) = e_x x + e_y y$. In this case, the directionality $D$ can be rewritten as

$$D = \frac{2 \text{Im} \left\{ e_x^* e_x \right\}}{|e_x|^2 + |e_y|^2},$$ \hspace{1cm} (6)

Note that the expression of (6) is the same to the definition of Stokes parameter $S_3$ for guided modes [39]. $S_3 = \pm 1$ is the circular-polarized point (C-point) that the local polarization in this point is right-handed circular polarized (RCP) or left-handed circular polarized (LCP). With placing a chiral dipole emitter in one of those C-points, we can implement UDC with $D = \pm 1$, regardless of the coupling of other modes. More details will be confirmed by the simulation results in section IV.

With regard to the effect on the coupling of other modes, another quantity $\beta$ should be introduced to describe the coupling efficiency for rightward or leftward waves. $\beta$ is defined as the ratio between the total emission power $P_{\text{tot}}$ and the power coupling to objective direction ($P^R$ or $P^L$). For example, as a target for rightward UDC, we can place a $\mathbf{d}$, dipole emitter at the RCP C-point of rightward guided mode. Thus the leftward wave is completely suppressed that $P^L = 0$. In this case, the rightward UDC efficiency $\beta^R$ is expressed by

$$\beta^R = \frac{P^R}{P_{\text{tot}}},$$ \hspace{1cm} (7)

In order to obtain high-efficiency UDC, we expect to improve the optical power output from rightward direction, where $P^R$ at C-point is proportional to the in-plane electric field intensity $|e_x|^2 + |e_y|^2$. In other word, it is important for the match condition between C-points and power peak of in-plane electric
fields, which is accessible from glide-plane symmetry [14], [40].

III. TOPOLOGICAL STATES IN VALLEY PHOTONIC CRYSTALS AND THEIR INTERFACE WAVEGUIDES

The main target of this work is to discuss the unidirectional coupling (UDC) behavior in valley-interface photonic crystal waveguides (VPCWs). The interfaces are formed by two topologically-distinct domain walls, consisting of honeycomb-lattice photonic crystals (PCs). As shown in Fig. 1(a), the unit cells of honeycomb-lattice PCs contain two air holes in dielectric slab ($\varepsilon = 12$) with thickness $h = 0.6a_0$. $a_0$ and $r_0$ are lattice constant and initial radius of air holes, respectively. $\delta$ is a detuning parameter of radii $r_1$ and $r_2$ between two air holes, where $r_1 = r_0(1+\delta)$ and $r_2 = r_0(1-\delta)$. Here we focus on the TE-polarization photonic bands and their eigenmodes, calculated by MIT Photonic Bands (MPB) package [41]. Consider $\delta = 0$ that two air holes are equivalent, gapless Dirac-cone dispersion can be observed near K/K’ points derived from the $C_n$ symmetry [see black dashed line in Fig. 1(c)]. For non-zero detuning parameter that $\delta \neq 0$, the degenerate points at K’ are lifted [red and blue solid lines in Fig. 1(c)] and opened a TE-like bandgap. Fig. 1(b) depicts the phase map of bandgap size varying with $r_0/a_0$ and $\delta$. The larger $\delta$, the larger gap size. We chose A-type VPC ($r_0 = 0.22a_0$ and $\delta = 2/11)$ and B-type VPC ($r_0 = 0.22a_0$ and $\delta = -2/11$) in the following discussion, highlighting as blue and red dots with the same gap size ~8.32%. Note that the phase map only shows the results of hole-type interfaces, i.e. the region for $2r_0^i(1+|\delta|) \leq a_0/\sqrt{3}$.

The above PCs attribute for valley-Hall topological phases so that we call them valley photonic crystals (VPCs). In Fig. 1(c), A-type and B-type PCs share the same band structure, since they are $y$-axis mirror symmetry partners with each other. But their topological invariants are inequivalent. The topology of PCs can be characterized by valley Chern index $C_V = C_K - C_K'$, where $C_K$ or $C_K'$ is the index integrating Berry curvature over the K- or K’-valley domain. Note that Berry curvature can be calculated by $\Omega = i \nabla \times (\psi \nabla \psi^\dagger)$, where

$$\psi(x, y, z) = \left[\sqrt{\vare}_E(x, y, z)u^E_n(x, y, z), \sqrt{\vare}_H(x, y, z)u^H_n(x, y, z)\right]^T$$

is the eigenfield retrieved from MPB. Due to bulk-edge correspondence, the topology of TE-like gap $C_V^{gap}$ depends on the summary of valley Chern indices of TE-like bands $C_V^i$ below such gap, i.e. $C_V^{gap} = n \sum C_V^i$, where $n$ is the TE-like band index below the gap. In regard to this work, the TE-like gap for our case only depends on the topology of the first TE bulk band. An intuitive result is to see Berry curvature distribution of the first TE band for two types of PCs, as shown in Fig. 1(d). For A-type VPC, the peak of Berry curvature is mainly localized around K valley while sink is around K’ valley. By contrast, B-type VPC reverses the sign of Berry curvature distribution. As a consequence, the sign of the valley Chern index is the same to the sign of detuning parameter $\delta$, i.e. $C_V > 0$ for A-type VPC with $\delta > 0$ while $C_V < 0$ for B-type VPC with $\delta < 0$. Such two type VPCs are topologically distinct.

Valley-dependent topological edge states can be constructed in the interface of VPCs with different valley Chern indices. Here we discuss two types of typical interfaces along $\Gamma$K direction, i.e. bearded-stack and zigzag-stack VPCWs. Although the UDC in these two VPCWs is intrinsic, the performance is different. For bearded-stack configuration shown in Fig. 2(a), such interface possesses glide-plane symmetry, i.e. the structure is invariant under a mirror operation along $y$ axis and then a translation along $x$ axis with half lattice constant. On the other hand, the zigzag-stack VPCW is inversion symmetric along $y$ axis [see the configuration of Fig. 2(b)].

Edge dispersions for both types of VPCWs are shown in the red lines of Fig. 2(a) and Fig. 2(b). Protected by TRS, a pair of counter propagation modes exists together. Gray regions and purple regions are the light cone of air and the projected bands of the bulk states, respectively. To connect the eigenmode analysis to the full-wave simulation, we consider the region where only a pair of guided modes exists, the so-called single-mode region. The single-mode regions of bearded and zigzag VPCWs are marked as yellow and green area, respectively. Note that the single-mode bandwidth of zigzag VPCW (~7.68%) is less than that of bearded VPCW (8.32%), but both of them are large enough to implement broadband UDC.

IV. DIRECTIONALITY AND UNIDIRECTIONAL COUPLING EFFICIENCY ANALYSIS

Next, we will discuss the UDC performance in bearded- and zigzag-stack VPCWs, in terms of both directionality $D$ and UDC efficiency $\beta$. According to (6), the Stokes parameters $S_3$ of rightward eigenmode is equal to the directionality $D$ of light propagation, if only guided modes are excited by a dipole emitter. Here the eigenmodes of VPCWs are their edge states. Marked as ‘I’ and ‘II’ in the single-mode regions of Fig. 2(a) and Fig. 2(b), we choose such two edge states at the frequency $0.304 / a_0$ to discuss the unidirectional coupling behaviors, since both two modes operate below light cone. Fig. 2(c) is the distribution of Stokes parameter $S_3$ at $z = 0$ plane, calculated from the in-plane electric fields ($E_x$, $E_y$) of eigenmode at ‘I’ and ‘II’ points. $S_3 = 1$ and $S_3 = -1$ mean local RCP and LCP, respectively. When a chiral dipole emitter is placed in one of those C-points, the excitation fields will only couple to rightward or leftward guided mode and thus propagate along $+x$ or $-x$ direction, regardless of the coupling of other waves.

For confirmation, the finite-difference time-domain (FDTD) simulations have been performed in the finite-length VPCWs. The length is $31 a_0$ along $x$ axis here. The chiral point emitter in simulation is synthesized by two linear-polarized dipoles with the same amplitude at the same position. One is $E_x$-polarized dipole with zero initial phase, and the other is $E_y$-polarized dipole with $\pm \pi/2$ initial phase. A monitor plane perpendicular to $x$ axis is placed at the right (left) end of VPCWs to collect the rightward (leftward) optical power $P_R$ ($P_L$). When a chiral dipole emitter is placed in a certain location $r$, the directionality

$$\delta = \epsilon \mu \frac{x}{\varepsilon}$$

$$\phi$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JLT.2020.3024696, Journal of Lightwave Technology
$D(r)$ of light propagation can be retrieved from the simulation results of $P^R$ and $P^I$ based on (5). Figure 2(d) gives the directionality profiles for a $d$, dipole at various location $r'$ but fixing at $z = 0$ plane. The operation frequency 0.304 $c/\lambda_0$ is consistent with the eigenmode at 'I' and 'II' points. We can see that the directionality profiles by full-wave simulations are very similar with the distribution of Stokes parameter from eigenmode calculations, in correspondence with the theoretical prediction by (6). Such eigenmode analysis method provides a simple guideline to predict the coupling behavior between chiral dipole emitters and VPCWs.

Note that these two profiles are not exactly the same with each other, because i) the waves propagate along finite-length VPCW in the full-wave simulation model while the eigenmode calculations is periodic along $x$ axis (in corresponding to infinitely long), ii) the full-wave simulations will excite other modes beyond the guided eigenmodes in a manner. For reason i), we can increase the number of VPCW unit cells in full-wave simulations to approach the periodic boundary of eigenmode calculations. For reason ii), the unidirectional coupling efficiency $\beta$ should be introduced to evaluate UDC performance, combined with the directionality $D$. In the following discussion, we will take rightward UDC case as an example. According to Eq. (7), $\beta^R$ is defined as $P^R$ over $P^{tot}$, where $P^{tot}$ is total optical power coupling for all modes. In full-wave simulations, it is easy to obtain $P^{tot}$ by adding a box of power monitor covering the dipole emitter. At those near-unity-$D$ points, the larger in-plane electric field intensity is, the higher $\beta^R$ will be excited in principle. The inversion symmetry of PCW will ensure the intensity peak localized at the center of interface, which is always a manifestation of linear polarization. This issue can be overcome by constructing glide-plane symmetry PCW.

Fig. 3(a) show the simulated profiles of rightward UDC efficiency $\beta^R$ for bearded-stack VPCW and zigzag-stack VPCW, excited by $d$, dipole emitter at various location $r'$ but fixing at $z = 0$ plane. The operation frequency is 0.304 $c/\lambda_0$. The five nearest-unity-$D$ points of Fig. 2(d) are labelled as pink dots in Fig. 3(a). Particularly, four of the five nearest-unity-$D$ points are situated in the interface hole of bearded-stack VPCW. We also plot the UDC efficiency curves in Fig. 3(b), when the dipole emitters are placed at the five nearest-unity-$D$ points. Their UDC efficiency are quite different in these point.

Then the upper one is shifted half lattice constant along $x$ axis and thus the bulk PC is divided into upper and lower domain walls. The LPCW is constructed by triangle-lattice PC with $a_1$ lattice constant of triangle-lattice PC. To form a channel that supports 1D propagating defect mode, the interface of the two domain walls can be taken as the air hole slab in the background of dielectric slab. The thickness of slab is the same as that of VPCW. Radius of air holes is 0.3$a_1$, where $a_1$ is the lattice constant of triangle-lattice PC. To form a channel that supports 1D propagating defect modes, all the air holes are removed along $x$ axis and thus the bulk PC is divided into upper and lower domain walls. Then the upper one is shifted half lattice constant along $x$ axis to ensure glide-plane symmetry, as well as zooms in $\sqrt{3}a_1/6$ width along $y$ axis for 120°-bending construction [see Fig. 4(g) for the structure illustration]. In other word, the width between the air-hole centers of the first two rows decreases to be $5\sqrt{3}a_1/6$, while conventional W1 PCW is $\sqrt{3}a_1$. Fig. 4(d) confirms that high-efficiency UDC can be also realized in straight LPCW at the frequency 0.272 $c/\lambda_1$. But such high directionality fails to maintain in $\Omega$ shape LPCW as shown in Fig. 4(e). The backward waves are comparable with the original forward waves because of the strong reflection around the corners. There is a few of frequency reaching near-unity $D$ in the directionality spectra of Fig. 4(f), in contrast with broadband unidirectional coupling of VPCW.

Note that one can suppress the bending reflection in LPCW by use of topology optimization [42]. In this way, we would obtain broadband UDC along sharp-bending interface with a local method, i.e. finely engineering the local geometry around...
the bending corner. Different from the local method in LPCW, topological waveguides (e.g. VPCW in this work) provide a global method to achieve broadband robustness. The global method means such robustness is one of the intrinsic properties of VPCWs, which could be totally predicted by investigating the bulk topology, regardless of the local region around corner. Therefore, some promising applications for light manipulation in topological nanophotonics (e.g. UDC) will benefit by the decrease of design complexity.

VI. CONCLUSION

In summary, we have established an eigenmode-analysis model to predict the unidirectional coupling performance in valley photonic crystal waveguides, as well as been confirmed by full-wave simulation results. It provided a guidance that near-unity directionality could be obtained when the chiral dipole emitter was placed at the C-point. We also found that bearded-stack VPCW with glide-plane symmetry manifested higher UDC efficiency than the zigzag-stack VPCW with inversion symmetry. Due to topological protection, the UDC in VPCWs supported robust transport against sharp-bending interface. The topological phase in valley photonic crystals gives an intrinsic and deterministic method to design high-efficiency chiral interface supporting robust transport, which is a new paradigm to explore chiral light-matter interaction. The guidance of our work can develop chiral topological photonics into potential applications for complex information processing in chiral nanophotonics [2] and chiral quantum optics [3], such as spin-path coding and spin-dependent splitter.

Despite the UDC, we believe the fast development of topological integrated photonics will render many opportunities from fantastic physics, novel integrated devices and its system-on-a-chip in future. Firstly, integrated photonics is a standard platform of nanofabrication and measurement to explore the intrinsic features of topological physics, which are difficult to implement in condensed-matter systems. For example, it is accessible to retrieve topological photonic bands and invariants, by using angle-resolved spectrometer to reveal the evolution of radiated far fields at k space. Secondly, topological photonics provides a new paradigm to precisely control light-matter interaction based on topological modes, particularly at visible wavelength. In order to achieve high-performance topological photowaveguides at visible range, a proper platform with high-quality thin film is required to overcome the trade-off between absorption loss and refractive index. Thirdly, one of promising applications is to design a topological nanophotonic or quantum system on a single chip, by integrating different types of topological photonic devices. The key issue may be the compatibility of different kinds of topological phases in the same platform.

REFERENCES


[26] X. Chen, F. Zhao, M. Chen, and J. Dong, “Valley-contrast physics in all-dielectric photonic crystals: Orbital angular momentum and


Fig. 1. Topologically optical properties of bulk states in valley photonic crystal (VPC) slab. (a) Unit cell sketch of VPC in honeycomb lattice, consisting of two air holes in dielectric slab (\( \varepsilon = 12 \)) with thickness \( h = 0.6a_0 \), \( a_0 \) and \( r_1 \) are lattice constant and initial radius of air holes, respectively. \( \delta \) is a detuning parameter of radii \( r_1 \) and \( r_2 \) between two air holes, where \( r_1 = r_1(1+\delta) \) and \( r_2 = r_2(1-\delta) \). Based on the sign of \( \delta \), two types of VPC can be configured as shown in red and blue cases. (b) Phase map of VPC gap size varying with \( r/a_0 \) and \( \delta \). The larger \( \delta \) the larger gap size. We chose A-type VPC \((\delta = 0.22a_0 \) and \( \delta = 2/11 \)) and B-type VPC \((\delta = 0.22a_0 \) and \( \delta = -2/11 \)) in the following discussion, highlighting as blue and red dots with the same gap size - 8.32%. Note that only the parameters of hole-type edges are shown in the map. (c) Bulk band structures of TE-like polarization with \( \delta = 0 \) (black dashed), \( \delta = 2/11 \) (red solid) and \( \delta = -2/11 \) (blue solid). For \( \delta = 0 \), gapless Dirac-cone dispersion can be seen near K/K' points derived from the Cau, symmetry of unit cell. For non-zero detuning parameter \( \delta \neq 0 \), the degenerate points at K/K' are lifted and opened a topologically nontrivial bandgap. Note that A-type and B-type VPCs share the same band structure, since they are y-axis mirror symmetry partners with each other. (d) Berry curvature distribution for two types of VPCs. For A-type VPC, the peak of Berry curvature is mainly localized around K valley while sink is around K' valley. By contrast, B-type VPC reverses the sign of Berry curvature distribution. Thus A-type and B-type VPCs have different sign of valley Chern indices, indicating they are topologically distinct.
Thus we can see the UDC efficiency of bearded VPCW is superior to that of zigzag interface. (c) UDC efficiency spectra as a function of normalized frequency, when the emission dipole \( d \) are placed at circle pink dots (index 5) of the insets. The UDC of bearded VPCW (solid line) maintains a high-efficiency plateau over the single-mode region (yellow box for bearded-stack VPCW and green dashed box for zigzag-stack VPCW), in contrast with zigzag VPCW (dashed line). The strategy of bearded-stack VPCW provides us a simple guideline toward high-efficiency UDC, relaxing the bias error of frequency and source location.

Fig. 2. Edge states and directionality analysis in two type of valley-interface photonic crystal waveguides (VPCWs). Left panel: bearded-stack VPCW with glide-plane symmetry. Right panel: zigzag-stack VPCW with y-axis inversion symmetry. Such two interfaces are constructed by A-type and B-type valley photonic crystal (VPC) slab with opposite sign valley indices. (a-b) Schematic view and edge dispersions (red lines) for bearded and zigzag interfaces. Gray regions and purple regions are the light cone of air and the projected bands of the bulk states. The yellow box ranging from 0.288 \( c/a \) to 0.313 \( c/a \) indicates the single-mode region of bearded edge state, while the single-mode region of zigzag edge state is labelled as green box at the frequency interval between 0.288 \( c/a \) and 0.311 \( c/a \). Markers I and II represent the frequency of 0.304 \( c/a \), of which we will discuss the directionality below. (c) Stokes parameter \( S_0 \) profiles retrieved from eigenmode calculations and (d) directionality \( D \) profiles retrieved from full-wave simulations by selective excitation of \( d \), dipole emitter, both at the frequency of 0.304 \( c/a \) in two type of valley-induced interfaces. The \( S_0 \) profiles focus on \( z = 0 \) plane of eigenmode VII. Note that the directionality profiles are very similar with the distribution of Stokes parameter, in correspondence with our theoretical predictions.

Fig. 3. Analysis of unidirectional coupling efficiency in two type of valley-interface photonic crystal waveguides (VPCWs). (a) Profiles of unidirectional coupling (UDC) efficiency \( \beta \) for bearded-stack VPCW and zigzag-stack VPCW, excited by \( d \), dipole emitter at various location \( r' \) but fixing at \( z = 0 \) plane. The operation frequency is 0.304 \( c/a \). The five nearest-unity-\( D \) points of Figure 2D are labelled as pink dots here. For bearded-stack VPCW, four of the five nearest-unity-\( D \) points are situated in the interface hole. (b) UDC efficiency curve when the dipole emitters are placed at the five nearest-unity-\( D \) points. Their UDC efficiency are quite different in these points due to the effect on the coupling of other modes, although their directionality is extremely high. To achieve high UDC efficiency, glide-plane symmetry can ensure the match between C-point and field maxima to enhance the coupling of guided modes.

Fig. 4. Comparison of unidirectional couple (UDC) behaviors between (a-c) bearded-stack valley-interface photonic crystal waveguide (VPCW) and (d-f) line-defect photonic crystal waveguide (LPCW). Both two waveguides have glide-plane symmetry. (a-b) Electric field intensity excited by \( d \), dipole (pink ring) and \( d \), dipole emitter. Broadband high directionality can be obtained in the single-mode region for both straight and sharp-bending interfaces due to the suppression of inter-valley scattering. (c-f) A case-control study on topologically-trivial LPCW with glide-plane symmetry. The electric field intensity is excited by \( d \), dipole at the frequency 0.272 \( c/a \). Due to glide-plane symmetry, high-efficiency unidirectional coupling can be also realized in straight LPCW, but such high directionality fails to maintain in sharp-bending case because of the strong reflection around the corners. There is a few of frequency reaching near-unity \( D \), in contrast with broadband unidirectional coupling of VPCW. (g) Illustration of the structure around the corner based on a 120°-bending LPCW.

Wen-Sheng Ruan was a postgraduate in Sun Yat-sen University and has received the master degree now.

Xin-Tao He is now the Associate Research Fellow in Sun Yat-sen University. He engages in the fundamental research of topological photonics and their applications for integrated photonics and nanophotonics.

Fu-Li Zhao is now the Professor in Sun Yat-sen University. Her research interest covers ultrafast laser spectroscopy and dynamics measurements of novel photonic material and devises.

Jian-Wen Dong is now the Professor in Sun Yat-sen University. He received the B.S. and Ph.D. degrees in School of Physics and Engineering from Sun Yat-sen University, Guangzhou, China, in 2003 and 2007, respectively. He was a visiting scholar at University of California, Berkeley (UCB) and The Hong Kong University of Science and Technology (HKUST). His research focuses on the fundamental physics and optical information applications of topological photonics, nanophotonics, photonic crystals and metasurfaces. So far, he has published more than 50 scientific papers in the above fields and three of them are selected as ESI papers in September 2020.