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*Image:* Ornamental multiplication of space-time figures of temperature transformation rules (adapted from T. S. Bíró and P. Ván 2010 EPL 89 30001; artistic impression by Frédérique Swist).
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Electromagnetic Bloch-like oscillations in one-dimensional quasicrystal consisting of negative permeability metamaterial

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Abstract – Electromagnetic oscillations have been investigated in one-dimensional quasicrystal constructed by dielectric and negative permeability multilayers. Bloch-like oscillations are found in either double-period or Fibonacci quasiperiodic structure, even when no thickness gradient is introduced in the dielectric layer. The average temporal periodicity of the oscillation can be predicted by frequency spectrum with either a Wannier-Stark ladder profile or with a particular profile. Various characteristics are studied, such as electric field pattern of the scattering states and transmission behaviors in both frequency and temporal domain.

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Introduction. – Bloch oscillation is a fundamental phenomenon of charge transportation [1]. In the absence of dephasing processes, an electron in a periodic potential is accelerated in energy with the help of an external homogeneous static electric field, and subsequently undergoes a coherent periodic motion across the electronic lattice in real space. Similar nature between an electron in a solid state system and a photon in a dielectric medium inspires us to investigate the possibility of Bloch oscillation in photonic crystals. Recently, behaviors on Bloch oscillation have been found in various kinds of periodic systems [2–10], such as laterally confined Bragg mirrors [2], metal heterowaveguide superlattices [5], one-dimensional (1D) metamaterial-based photonic crystals [6,10], two-dimensional phononic crystals [9], binary waveguide arrays [8], and so on.

Quasicrystals (QCs) are known to possess long-range translational order, and they can be considered as intermediates between periodic and disordered systems [11]. QCs of 1D type, of which the sequences are described in terms of peculiar recursive relations, are interesting in many aspects. Wave transport and localization studies in electronic and photonic systems have been well studied (see, e.g., ref. [12–14]). A forbidden frequency region similar to photonic band gap has been found in Fibonacci QC [15,16]. In particular, electronic Bloch-like oscillation (BLO) has been studied in electronic QC [17–19]. The purpose of this paper is to study the BLO in the photonic QC system.

In this paper, the 1D QC that is constructed by dielectric and negative permeability layers is investigated. The photonic BLO is both found in a double-period and a Fibonacci structure. A distinct feature is that the thickness of the dielectric layer obeys the quasiperiodic recursion and it is no longer monotonic decreasing. The temporal periodicity of the BLO can be predicted by the frequency transmission spectrum with a unique profile.

Physical structure. – We will firstly study the double-period QC structure, with the recursion relationship $S_j = \{S_{j-1}, S_{j-1}^+\}$ and $S_j^+ = \{S_{j-1}, S_{j-1}\}$ for $j \geq 1$, with $S_0 = A$ and $S_0^+ = B$. Here $j$ is the generation number of the double-period structure. It is invariant under the transformations $A \rightarrow AB$ and $B \rightarrow AA$. The schematic diagram of the fifth-generation double-period structure is shown in fig. 1. The structural expression is ABCBABCBCBABCBCB, where $C = 3A$. The permittivity, permeability, and thickness of layer A are $\varepsilon_A = 3.0$, $\mu_A = 1.0$, and $d_A$, respectively. Layer B is a dispersive negative-permeability metamaterial with the permittivity and thickness of $\varepsilon_B = 2.5$ and $d_B$. The permeability obeys the Drude model with the form of $\mu_B = 1.0 - 120/f^2$, where $f$ is the working frequency of the incident electromagnetic (EM) wave measured in gigahertz. The negative-permeability metamaterial may
be potentially realized in artificial designed transmission lines [20]. For simplification purpose, we assume that the negative-permeability metamaterial is lossless.

As we know, in order to realize photonic Bloch oscillation in periodic EM system, one needs to construct EM Wannier-Stark ladder in frequency domain. The EM Wannier-Stark ladder can be realized by breaking the symmetry of multiple cavities, and subsequently the defect states in the adjacent cavities must be different from each other [3]. One of the most common methods is to use a series of coupled microcavities in which their cavity thicknesses decrease monotonously [4–6]. However, we will demonstrate that the QC system in which the cavity thicknesses are no longer monotonic decreasing can indeed generate a unique frequency spectrum that can also lead to BLO behavior, similar to the fact that the Wannier-Stark ladder can result in Bloch oscillation. Without loss of generality, we assume the thickness of layer B is a constant of \( d_B = 3 \) mm, and the thickness of layer A obeys the following expression: \( 1/d_{n+1} = 1/d_n + \delta/d_1 \), where \( d_n \) is the thickness of the \( n \)-th layer A, and \( \delta \) is the gradient number. We will show below that the BLO behavior can occur even when \( \delta = 0 \), i.e. no gradient is in the QC structure.

**Results and discussions.** – Figure 2 shows the characteristics of BLO in the fifth-generation double-period structure. Figure 2(a) shows the transmission spectrum as a function of frequency with the values of \( d_1 = 28 \) mm and \( \delta = 0.15 \). One can see that a series of equidistant transmission peaks in the frequency range from 6 to 8.5 GHz. This is the result of the coupling between the multiple microcavities in the QC. Interestingly, the frequency distances between adjacent peaks are almost the same with the value of around 0.282 GHz. This is very similar to those of structures with monotonic decreasing cavity thicknesses [4–6]. We expected that there is a sequence of strongly localized states present in the structure. This is demonstrated by the electric field distributions of the scattering states calculated by transfer matrix method [21]. In the calculation, a plane wave is normally incident from the left side of the structure. The electric field distribution for one frequency is firstly calculated by the reflection and transmission coefficients of the system. Then we change another frequency to calculate again. Finally we plot the scattering states map in the plane of frequency and position along the \( z \)-direction. The results are shown in fig. 2(b). The left panel shows the illustration of the double-period structure, and the right panel shows the scattering states map inside the structure in the frequency range from 6.5 to 9 GHz. The brighter color indicates the location of confined energy. There are four distinct localized states in the frequency range from 7.0 to 8.0 GHz, which is consistent with the transmission peaks as shown in fig. 2(a).

In order to study the temporal dynamics evolution of a light pulse inside the QC, we use a Gaussian pulse in the frequency domain with the expression of \( g(f) = (1/\pi \Delta f) \exp[-((f-f_0)/\Delta f)^2/2] \), where \( f_0 = 7.341 \) GHz is the central frequency of the localized scattering states.
and $\Delta f = 0.282\, \text{GHz}$ is the frequency distance between the adjacent localized scattering states. The profile of the Gaussian pulse is plotted by red curve in fig. 2(a). The time-dependent electric field distributions inside the structure can be calculated by scattering state theory [22], and the energy output from the structure can be calculated as the time-resolved transmission. The time-dependent electric field distribution is shown in fig. 3(a). Brighter color represents stronger intensity. The double-period structure is also illustrated in the upper panel of fig. 3(a). We can see that the incident Gaussian pulse starts at $t = 0$, and oscillates through the QC with a temporal periodicity of around $3.525\, \text{ns}$. In the theory of electronic Bloch oscillation, the temporal periodicity is inversely proportional to the energy difference between the neighboring localized states. So the theoretical periodicity of the studied structure can be predicted as $1/\Delta f \sim 3.546\, \text{ns}$, which is consistent with the simulation results in fig. 3(a). In addition, the intensity becomes smaller near $t = 12\, \text{ns}$ due to the wave propagating leakage. The time-resolved transmission spectrum is another feature of the BLO behavior. The results of fig. 3(b) show the unique oscillation characteristic.

The parameter $\delta$ is a gradient number which should not vanish in the periodic structure. However, this is not the case of the QC. When $\delta = 0$, the BLO phenomenon can still occur. Figure 4 shows an example of the double-period structure when the value of $\delta$ reaches zero. The permittivity and permeability indices are the same as before. The thicknesses of layer A and layer B are changed to $11$ and $3\, \text{mm}$, respectively. The frequency spectrum of this double-period structure is illustrated in fig. 4(a) from $3.5$ to $5.5\, \text{GHz}$. There are a localized state with single peak at the center ($\sim 4.654\, \text{GHz}$) and two localized states with multipeaks at both sides. When a Gaussian pulse with the central frequency as $4.654\, \text{GHz}$ and the frequency width as $0.44\, \text{GHz}$ is incident onto the structure (red curve in fig. 4(a)), the BLO behavior appears. The temporal dynamics results are shown in fig. 4(b). The average periodicity of the BLO is approximate to $4.01\, \text{ns}$. The periodicity with the value of $3.344\, \text{ns}$ can also be evaluated by the profile of transmission spectrum. It deviates slightly from the simulation results. This is because the transmission profile is somewhat different from an ideal Wannier-Stark ladder.

We also demonstrate the BLO in the QC arranged in Fibonacci lattice under the condition $\delta = 0$. We take the sixth-generation Fibonacci structure for example. The Fibonacci structure is constructed by two layers A and B according to the recursion relation, $S_{j+1} = \{S_j, S_{j-1}\}$, $S_1 = A$, $S_2 = AB$, where $j$ is the generation number of the Fibonacci unit cell. There are $F_j$ layers in $S_j$, where $F_j$ is a Fibonacci number given recursively as $F_{j+1} = F_j + F_{j-1}$, for $j \geq 1$, with $F_0 = F_1 = 1$. The sixth-generation Fibonacci structure is $ABAABABAABABAB$, and $F_6 = 13$. The permittivity and permeability indices are the same as before. The thicknesses of layer A and layer B are changed to $18$ and $3\, \text{mm}$, respectively. We find that the frequency spectrum shown in fig. 5(a) is similar to that of fig. 4(a). When a Gaussian pulse with the central frequency as $7.353\, \text{GHz}$ and the frequency width as $1.644\, \text{GHz}$ is incident, the BLO behavior can still appear. The result is shown in fig. 5(b). The temporal periodicity of the BLO is about $1.06\, \text{ns}$. We also evaluate the periodicity equaling $0.88\, \text{ns}$ by the profile of transmission spectrum, which is approximate to the simulation results.

One of the purposes of this work is to find out a new frequency spectrum which can also lead to BLOs, when the system is lacking thickness gradient. We find that such frequency spectrum is a discrete set of localized states.
The characteristics of the fifth-generation double-period structure and the sixth-generation Fibonacci structure. The BLO has been found in the quasiperiodic recursive relationship and it is no longer alternated by a dielectric layer and a negative permeability behavior in the 1D QC systems. The structure is larger than in the gradient one.

Fig. 5: (Color online) Same as fig. 4 except for the sixth-generation Fibonacci structure with no thickness gradient.

Fig. 6: (Color online) Temporal oscillation period as a function of cycle number. Square, circle, and triangular symbols are corresponding to figs. 3, 4, and 5, respectively. The solid lines are used to guide the eyes.

with non-commensurate and non-equal spacing, whose profiles are similar to those of figs. 4 and 5. The mechanism of BLOs in the structure without thickness gradient is the multi-interference coupling between cavities arranged in QC sequence, which is similar to the structure with thickness gradient. The temporal periods of BLOs still have difference between two kinds of structures. As shown in fig. 6, the temporal oscillation periods of no-thickness-gradient structures (circle and triangle) are more fluctuant than that with gradient (square), showing that the deviation of the average temporal periods in the no-gradient structures is larger than in the gradient one.

**Conclusion.** – In summary, we have investigated the BLO behavior in the 1D QC systems. The structure is alternated by a dielectric layer and a negative permeability layer. The thicknesses of the dielectric layers obey the quasiperiodic recursive relationship and it is no longer monotonic decreasing. The BLO has been found in the fifth-generation double-period structure and the sixth-generation Fibonacci structure. The characteristics of the oscillation in both frequency domain and temporal domain are studied in microwave range.

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